Determining Characteristic Mass for Low-Earth-Orbiting Debris Objects

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The increased number of man-made debris-producing events over the last several years has expanded the debris-size low-Earth satellite population enough to yield a finite probability of hypervelocity collisions over satellite lifetimes. Since the destructive capability of a hypervelocity collision is a strong function of the mass ratio between the two objects, the ability to withstand a collision of given mass becomes a good measure of survivability and shielding requirements for any space vehicle. Most of these particles, however, are at or below the threshold of detection and tracking with our current space tracking network. Several methods, both theoretical and empirical in nature, have been put forward in an effort to determine the size and mass of these debris particles. A technique that involves determining an accurate ballistic coefficient for near-Earth satellites is developed and used to estimate the remaining debris in orbit from one fragmentation event. The dependence of mass on radar cross-sectional measurements and particle-density estimates is also analyzed.

Introduction

As international space activity has increased, with its emphasis on long-term and even permanent manned space presence, the tracking of and protection against debris-size orbiting objects has received growing attention. Although the probability of collision with the current space population is acceptable, it will continue to increase into the next century and will be of legitimate concern to such projects as the international space station, Mir, and other operational and proposed manned and unmanned space platforms. Several efforts are ongoing in an attempt to quantify the near-Earth debris environment and its dangers, including those of Chobotov,[1] Kessler,[2] and McKnight and Brechin[3] to name just four.

Of particular concern are those space objects that are at or below the detection threshold of the current space-surveillance resources. Even though these objects typically have very small characteristic dimension, they pack a powerful and potentially damaging punch. A stray piece of aluminum 1 cm in diameter in low-Earth orbit is operationally undetectable and, yet, carries as much kinetic energy as a typical hand grenade. Very few of today's satellites could withstand a collision of that magnitude.

In fact, since the kinetic energy associated with a particle is a function of its mass and velocity, mass becomes a good measure of the destructive capability of any orbiting object with known altitude. Mass determination for small orbiting particles, however, is something less than an exact science. Methods of determining mass from physical size have been developed[4] but can only be used as a statistical prediction of mass, particularly in the small particle regime. The problem lies both in the process of determining radar cross section and in the probable composition of the debris-size satellites.

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There is one orbital characteristic that can be calculated with some success. This is the ballistic coefficient that determines the atmospheric drag force on the satellite. In the way of background, the importance of mass and size knowledge are discussed along with some of the current methods for determining these properties. A new method can then be developed using the calculated ballistic coefficient as the independent variable. The method is then applied to one satellite fragmentation event and the advantages and disadvantages are discussed.

Destructive Capability

One topic that gives rise to public debate and interest in the area of space debris is the subject of collision probability. There have been, and continue to be, studies that are designed to arrive at a probability of collision with a resident space object. One estimate puts the mean time between collisions with a 1-cm-size particle for an orbiter-size body at just over 10 yr and for a space station at 1 yr.[1] Observations made from a returned 1.5-m^2 thermal blanket from the Solar Max satellite yielded over 1000 measureable impact craters. Given, therefore, that on-orbit collisions are bound to occur, the next logical step is to determine the damage produced by the collision.

Several models exist for the prediction of catastrophic and noncatastrophic collisions and for the penetration depth of the impacting particle. Each of these models relies heavily on knowing the mass of the projectile. A test for the possibility of a catastrophic collision[4] is as follows:

$$M_2 < \Gamma M_1 \tag{1}$$

where Γ is the function of composition and velocity, M_1 the mass of projectile, and M_2 the mass of target. If the inequality in Eq. (1) is satisfied, the damage sustained will be enough to cause structural failure in the target. Estimates for spacecraft-type structures put Γ greater than 115.

Another way to measure the severity of collision is to calculate the depth of penetration that a given projectile will create when impacted with a target body. Two models include Bjork's equation[5]:

$$P = 1.09\,(MV)^{1/3} \tag{2}$$

where P is the depth of penetration, cm; M the projectile mass, g; and V the relative collision velocity, km/s; and Summer's equation:

$$P/D = 2.28\,(\rho_P/\rho_t)^{2/3}(V/C)^{2/3} \tag{3}$$

where P is the depth of penetration, D the projectile diameter, ρ_P the projectile density, ρ_T the target density, V the relative collision velocity, and C the speed of sound in target material.

The immediate observation can be made that severity of the collision damage will be a function of the projectile mass (knowledge of projectile density and diameter implies known mass). Since shielding requirements will be driven by the depth of penetration, tracking and measuring the mass of debris-size particles becomes an important field to research.

Measuring the Man-Made Satellite Population

Satellite Catalog

Primary responsibility for the detection and tracking of space objects rests with the United States Space Command (USSPACECOM) and its network of space-surveillance resources. This includes continuous maintenance of the Space Surveillance Center Catalog (SSCCAT) containing orbital element sets and information on the origin of all satellites currently in orbit and within the tracking capabilities of the network. The catalog is continuously updated and published monthly.

Currently, the SSCCAT contains information on over 8000 objects (only 5% of which are operational payloads). Since its inception the catalog has tracked over 20,000 objects, many of which have since re-entered the Earth's atmosphere. Of the 3700 payloads launched into space since the catalog has been maintained, approximately 100 satellite fragmentations have accounted for 50% of the total cataloged population. The detection limit of the network is on the order of 10 cm characteristic dimension, i.e., objects with a characteristic diameter of less than 10 cm are not detected. It has been estimated that there are 5–10 times the cataloged population that lie in the size regime of 1–10 cm characteristic diameter and, therefore, cannot be tracked. As mentioned earlier, these seemingly insignificant particles can still create catastrophic collisions simply due to the magnitude of the orbital velocities encountered (8–13 km/s). There are also some operational constraints on the system that result in an incomplete catalog including the requirement that any object be tracked by at least two tracking stations before cataloging, and the requirement (by United Nations treaty) that no satellite can be cataloged without knowing its country of origin.

Radar Cross-Section Catalog

Another duty of the USSPACECOM network is to provide a radar cross-section (RCS) catalog. This catalog is recorded and maintained by the AN/FPS-85 electronically steered phased array radar at Eglin Air Force Base, Florida. The monthly catalog gives a measurement of average radar cross-sectional area for that month. Most methods for determining mass of orbiting objects assume a knowledge of the physical size of that object. The RCS catalog represents the current extent of size predictions and, as such, should be examined in some more detail.

Determining Size

Unfortunately, the calculation of RCS has several inherent problems when being used for this type of analysis. The RCS is calculated from information on the radar itself and on the transmitted and incident electromagnetic fields. Radar cross section, therefore, is not directly measured and is sensitive to the radar and field characteristics used to calculate it. The monostatic radar equation can be used to demonstrate these sensitivities.

$$\sigma = \frac{PG^2\lambda^2}{(4\pi)^3(S/N)R^4BT\kappa} \tag{4}$$

where σ is the radar cross section, P the power, G the antenna gain, λ the radar wavelength, S/N the signal-to-noise ratio, R the range to target, B the bandwidth, T the loss temperature, and κ the Boltzmann's constant.

One of the most important aspects to RCS determination is its dependence on the square of the radar wavelength. The consequence of this fact is for different radars to find different values of radar cross section for the same object. Figure 1 shows the dependence of radar cross section on radar frequency (inverse wavelength). This figure also suggests that a radar can be optimized to measure a given size object by tuning the radar wavelength. This is the theory behind the development of a debris-dedicated radar now in development.[6]

The physics behind the wavelength dependence on detection capability is the Rayleigh region scattering phenomena, which introduces a significant error source as the wavelength approaches the object's characteristic dimension. This is the primary cause for the worsening accuracy of RCS for debris-size objects with the Eglin radar.[7] One possible way to minimize this error is to use an average RCS value from several radar resources. This, however, is not being considered in the operational sense to the knowledge of the authors.

Radar cross section also depends on the orientation of the object relative to the radar. An edge view of a flat plate will return a significantly lower value of RCS than if the plate is head on to the radar. The Eglin catalog reduces the RCS uncertainty for tumbling objects by introducing a weighted-averaging process to determine the RCS value entered into the RCS catalog.[8]

Mass Determination

Knowing how the size is determined and understanding its limitations now allows for a discussion of mass calculation. Currently, the most widely used method for converting size to mass is that developed by Kessler and Cour-Palais[4]:

$$M = 62 \times 10^3 (A_c)^{1.13} \tag{5}$$

where M is the mass of object, g; and A_c the cross-sectional area, m^2.

In this equation, 62×10^3 corresponds to the object density and 1.13 is a geometrical factor that represents a tradeoff between a hollow cylinder (1.0) and a solid cylinder (1.5). The conversion was found empirically against data from on-orbit payloads, empty rocket bodies, and the measurement of re-entered debris. The curve is shown in Fig. 2.

Since Eq. (5) is an empirical relationship, it cannot be relied upon to accurately predict mass for individual satellites. Also, as the only available cross section measurements are RCS val-

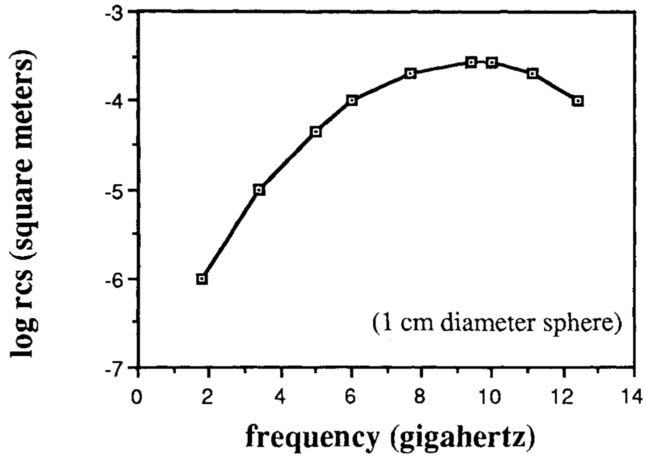

Fig. 1 Dependence of radar cross section on transmission frequency.[6]

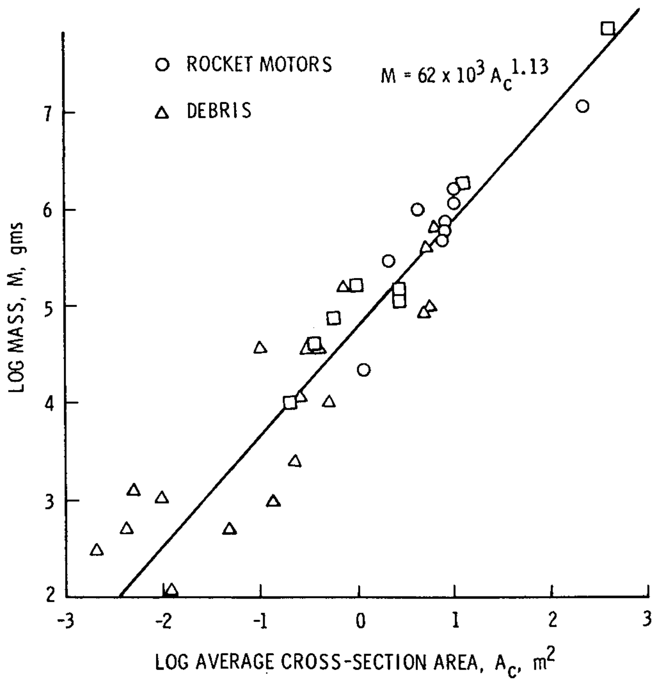

Fig. 2 Empirical relationship between mass and radar cross section.[4]

ues, the relationship carries the associated RCS error sources mentioned in the preceding section.

It was mentioned previously that the ballistic coefficient BC can be determined with relative accuracy. The BC can now be introduced to relax (but unfortunately not to eliminate) the dependence of mass calculation on RCS.

Use of the Ballistic Coefficient

For objects of known composition and geometry, the ballistic coefficient can be directly calculated.

$$BC = C_d A/M \tag{6}$$

where BC is the ballistic coefficient, C_d the drag coefficient, A the drag area, and M the spacecraft mass. For objects for which these parameters are unknown, no such calculation can be made. This is the case for all orbiting debris resulting from satellite fragmentations and when this information is unknown for any number of reasons.

Even though the ballistic coefficient itself cannot be measured, its direct effect on the object's orbit can. The atmospheric drag force is a function of BC [Eq. (7)] and causes the semimajor axis to decrease for satellites with trajectories within the sensible atmosphere.

$$F_d = \tfrac{1}{2} BC \rho V^2 \tag{7}$$

where F_d is the drag force per unit mass, ρ the atmospheric density, and V the spacecraft velocity. This "decay" can then be used to determine the ballistic coefficient with no independent knowledge of particle size and mass.

Methodology

The method behind the BC determination relies on the use of an orbit propagator and USSPACECOM's SSCCAT. The orbital elements of an object at some epoch time are fed into the propagator with an arbitrary estimate of the ballistic coefficient. The initial elements are taken from one month's SSCCAT. The propagator program is then allowed to run for a period of time long enough to notice drag effects. The predicted semimajor axis can then be compared to the SSCCAT's observed value at the output time. The process is then iterated by making new estimates of the ballistic coefficient. The tolerance level is set by the operator and was 0.5 km in semimajor axis over the 10–12 month propagation cycle used for this study.

Another advantage of using this long propagation time is the opportunity to use mean orbital elements. SSCCAT data are given in Kozai mean element form and, hence, can be used without transformation. Use of osculating elements to account for short-term periodic effects would be of minimal use since these variations average out over the course of propagation times reaching several months to a year. The propagator used is the long-term orbit predictor (LOP) written by Dr. Johnny Kwok of the Jet Propulsion Lab.[9] The primary modification to the code was the replacement of the propagator's exponential density model with a hybrid atmospheric model.

Since this analysis is based on the measurement of drag effects, it is important to include a high-fidelity atmospheric model. Diurnal effects and the variation of atmospheric heating due to the 11-yr solar cycle can yield significant temporal deviations in the local atmospheric density, translating into time dependence of the resultant drag force. The hybrid model consists of the Jacchia 1971 atmosphere programmed by Tobiska[10] and a tabular density model programmed by Kwok. In the Jacchia model's region of validity (120–550 km), it provides a dynamic density variation corresponding to the appropriate solar flux values over the propagation time. With accurate predictions of the atmospheric density and with velocity being known given orbit geometry and position, a certain amount of confidence can be associated with a ballistic coefficient calculated in this manner. Calculations using satellites of known geometry showed this methodology to yield a BC somewhat low but always within 25%.

As a note, the SSCCAT data format contains a field for a term called B^*, which is determined in much the same way as the method described previously. The static atmosphere and the short propagation times, however, introduce errors into the calculation of B^* that were mitigated in this analysis. Indeed, the B^* term can be anywhere from 20% to a factor of 30 in error. Hence, the B^* term is not used extensively in the NORAD analysis of the object.

Calculating Mass

An accurate ballistic coefficient does not give mass information, and at this point some assumptions must be made in order to determine object mass. The assumptions that must be made are regarding object shape and density. The assumed shape of the objects for this analysis is spherical in keeping with the definition of radar cross section. The RCS is used as a point of reference to make an estimate of object density. Given this estimated particle density and the object's ballistic coefficient, the mass can be calculated as follows:

$$M = \rho V = \rho(4/3)\pi r^3 \tag{8}$$

also,

$$M = C_d A/BC = C_d \pi r^2/BC \tag{9}$$

which yields two equations in the two unknowns M and r. Solving

$$M = 9\pi C_d^3/16\rho^2 BC^3 \tag{10}$$

The drag coefficient is a complicated function taking into account the interaction of the material and the local atmospheric environment. A commonly accepted value in a static atmosphere is 2.2 which, when corrected for an average velocity of the rotating atmosphere, yields the value of 2.0 used in this analysis.[11] Given the object's ballistic coefficient calculated using LOP, the mass can then be found after an estimate of the density is made. This in itself is a topic of some interest and is discussed in the following section.

Particle Density

Since any spacecraft is made up of several different materials, any debris that that spacecraft creates may also be of various composition. The primary component of satellite systems, however, is aluminum with a density of 2700 kg/m^3. It is on this density that the estimate needed for calculating mass with Eq. (10) is based.

Although it is always dangerous to generalize, it is held that smaller pieces will, on average, be of higher density than larger fragments from the same parent satellite. The reason for this comes from the physics of the actual breakup event. One would expect the small pieces to be solid fragments from a larger structure. The larger objects, however, may still be composed of several component parts with the accompanying voids scattered throughout the structure.

Using this trend and the scheme proposed by McKnight,[12] the function relating density to RCS was developed. The particles near the radar detection limit (RCS = 0.01 m^2) were given a density of solid aluminum and that density was allowed to decrease by a factor of two for every order of magnitude increase in RCS. This leads to the curve shown in Fig. 3, which obeys the relationship

$$\rho = 675.0\ (\mathrm{RCS})^{-0.301} \tag{11}$$

All of the components are now in place to estimate the mass of debris-size low-Earth objects.

Application

To determine the accuracy of mass determination using this method, one satellite fragmentation event was chosen for analysis. The event was the breakup of satellite 1979-17 (Solwind) as a result of the U.S. Anti-Satellite (ASAT) test program. The breakup occurred on September 13, 1985, and created 264 debris fragments that were entered into the SSCCAT prior to March 1988. Data was taken from the catalog at 6 and 18 months after the breakup and 107 pieces were analyzed. The remainder of the pieces had either not been cataloged as of March 1986, or had decayed as of March 1987. (It is not unusual for 10–20 pieces of 1979-17 to decay every 3-month reporting period.)

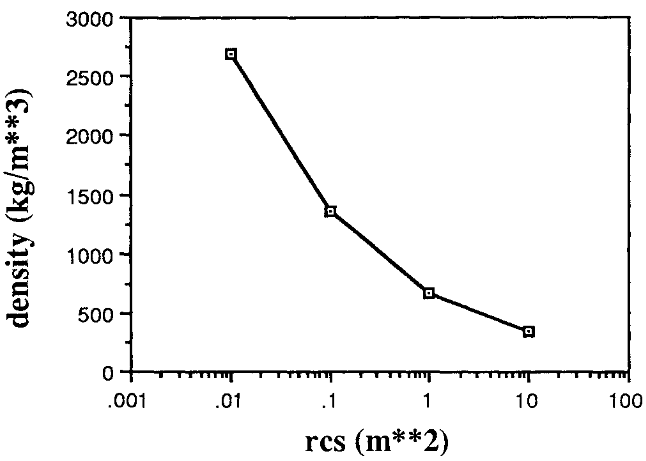

Fig. 3 Density function used in determining on-orbit mass.

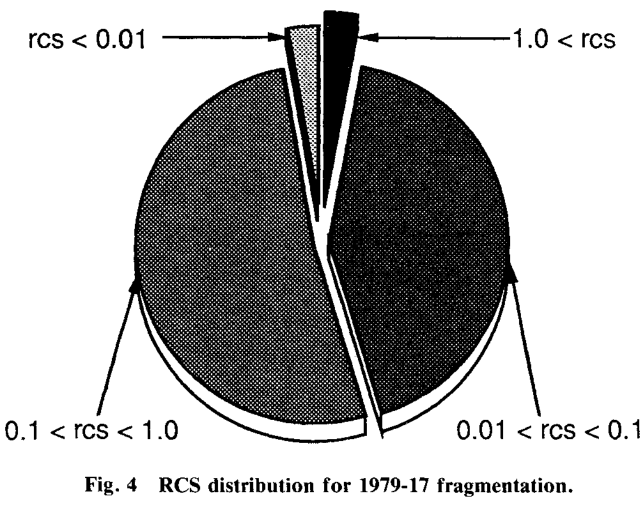

Fig. 4 RCS distribution for 1979-17 fragmentation.

This piece count represents 41% of the fragments from 1979-17 but most likely something less than 41% of the on-orbit mass. This is a result of USSPACECOM's inability to track very small pieces, of which there are probably many, and the likelihood that some mass deorbited prior to being cataloged (there can be a delay of several months before all pieces are cataloged and identified as belonging to a specific parent satellite). It would be expected that these pieces represent approximately 15–35% of the remaining 1979-17 on-orbit mass.

Figure 4 shows the distribution of RCS values for the 107 pieces of 1979-17 used in this analysis. A radar cross section of 0.01 m^2 is very near the detection limit for the radar resources and may explain the relatively low number of fragments cataloged in this region compared to what would be expected.

Using the density function given by Eq. (11) and the ballistic coefficients calculated using the orbit propagator, the on-orbit mass was determined to be 207 kg or approximately 28% of the 850-kg mass of the parent satellite. This figure is within the range estimated prior to the analysis. Of the 107 data points, one was discarded after showing a value for mass that was highly suspect. The piece in question (SSCAT #16311) maintained a mass greater than that of the satellite itself. The most likely explanation for this anomaly is the object's density being incorrectly assumed by the curve in Fig. 3. A reduction in the object's density quickly brought the mass into a more realistic range.

Summary

Because of the strong dependence on mass for damage assessment and shielding requirements for low-Earth satellites, the prediction of mass for trackable particles using existing information becomes important. By quantifying the orbital decay of these satellites, a ballistic coefficient can be calculated, which, when coupled with some knowledge of object shape and density, can be used in making a prediction of particle mass. Although the accuracy of a mass prediction made in this manner may be limited when analyzing specific objects, it can be used to provide information on a collection of particles or a range of particle sizes.

Conclusion

The results of this study tentatively sustain several hypotheses on the relationship between size and mass for debris-size objects in low-Earth orbit and serve to reinforce other work being done in this area. A higher degree of confidence in the methodology presented in this analysis will be obtained by extending the work to include additional satellite fragmentations and the various types of fragmentations experienced.

This analysis is a good indication of the problems inherent in determining size and mass for small space objects. The need for a more accurate characterization of the debris environment, in fact, has been voiced in both national and international forums and should be aggressively pursued in the years to come.

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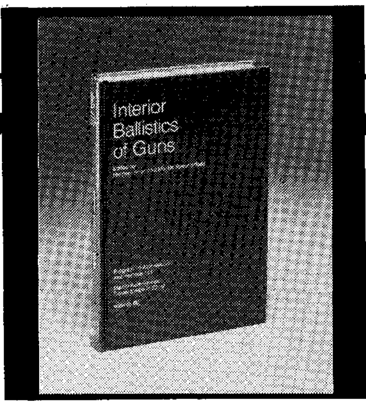

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